Statistics

Lecture 15

Feb 19-8:47 AM

S6.16

Binomial Prob. dist.:

1) we have n independent events (trials).

2) Each event has only two outcomes

$P(\text{Success}) = p$ $P(\text{Failure}) = q$

$p + q = 1$

$q = 1 - p$

3) p & q remain unchanges for all n trials.

4) $x \rightarrow$ # of Successes

$n - x \rightarrow$ # of failures.

$$P(x) = {}_nC_x \cdot p^x \cdot q^{n-x}$$

# of Combinations of x Successes in n trials.

Oct 27-10:36 AM

How many combinations of 2 Successes in 5 trials?

$_5C_2 = 10$

5 Math ▶ PRB ▼ nCr 2 enter

How many ways can we Select 4 people from a group of 10? $_{10}C_4 = 210$

10 Math ▶ PRB ▼ nCr 4 enter

Find $_{50}C_5 = 2,118,760$

50 Math ▶ PRB ▼ nCr 5 enter

Lotto

Select 5 numbers from 1 to 50.

Oct 27-10:42 AM

Consider a binomial Prob. dist. with

$n=5$, and $P=.6$ → $q = 1-P = .4$

Find $P(x=2) = {}_5C_2 \cdot (.6)^2 (.4)^3$

$x=2$

$n-x=3$

$P(x) = {}_nC_x \cdot p^x \cdot q^{n-x}$

$= 10 \cdot (.6)^2 \cdot (.4)^3$

^ ÷

$= .230$

Oct 27-10:48 AM

Consider a binomial Prob. dist. with $n=8$ & $p=.4$.

$q = 1-p = \boxed{.6}$ $\quad np = 8(.4) = \boxed{3.2}$

$npq = 8(.4)(.6) = \boxed{1.92}$

$P(x=3) = {}_8C_3 \cdot (.4)^3 \cdot (.6)^5 = \boxed{.279}$

$P(x) = {}_nC_x \cdot p^x \cdot q^{n-x}$

$= 56 \cdot (.4)^3 (.6)^5 =$

Oct 27-10:52 AM

Use binomial Prob. dist. with $n=64$ & $p=.5$

1) $q = 1-p = \boxed{.5}$ $\quad$ 2) $np = 64(.5) = \boxed{32}$

3) $npq = 64(.5)(.5) = \boxed{16}$ $\quad$ 4) $\sqrt{npq} = \sqrt{16} = \boxed{4}$

5) $P(x=35) = {}_{64}C_{35} \cdot (.5)^{35} \cdot (.5)^{29} = \boxed{.075}$

6) $P(x=60) = {}_{64}C_{60} \cdot (.5)^{60} \cdot (.5)^{4} = \boxed{3.4 \times 10^{-14}}$

Oct 27-10:58 AM

Suppose you are taking a multiple-choice exam with 100 True-False questions.

You are making random guesses.

1) $n = 100$ 2) $p = .5$ 3) $q = .5$

4) $np = 50$ 5) $npq = 25$ 6) $\sqrt{npq} = \sqrt{25} = 5$

P(guess exactly 60 correct answers)

$P(x = 60) = {}_{100}C_{60} \cdot (.5)^{60} \cdot (.5)^{40}$

$= .011$

P(guess correctly on all questions)

$P(x = 100) = {}_{100}C_{100} \cdot (.5)^{100} \cdot (.5)^{0}$

$\approx 7.9 \times 10^{-31}$

Oct 27-11:10 AM

Using TI:

2nd VARS ↓ binompdf(n, P, x enter

Consider a binomial Prob. dist. with $n = 5$ and $p = .6$

Trials:

p:

x-value

Paste Enter

No Menu

Find $P(x = 4) = \text{binompdf}(5, .6, 4) = .259$

Your work

Oct 27-11:18 AM

Consider a binomial Prob. dist. with $n=10$ and $P=.4$.

$P(x=5) = \text{binompdf}(10, .4, 5) = \boxed{.201}$

$P(x=8) = \text{binompdf}(10, .4, 8) = \boxed{.011}$

$P(x=5 \text{ or } x=8) = .201 + .011 = \boxed{.212}$

$P(x=0) = \text{binompdf}(10, .4, 0) = \boxed{.006}$

0 ↑ NO Successes

$P(x=10) = \text{binompdf}(10, .4, 10) =$

10 ↑ All Successes $\approx \boxed{1.05 \times 10^{-4}}$

Oct 27-11:22 AM

$P(x=a) = \text{binompdf}(n, P, a)$

$P(x \leq a) = \text{binomcdf}(n, P, a)$

Consider a binomial Prob. dist. with $n=64$ and $P=.5$.

$P(x=35) = \text{binompdf}(64, .5, 35) = \boxed{.075}$

$P(x \leq 35) = \text{binomcdf}(64, .5, 35) = \boxed{.809}$

$P(x<40) = P(x \leq 39) = \text{binomcdf}(64, .5, 39) = \boxed{.970}$

Oct 27-11:30 AM

Consider a binomial prob. dist. with $n=150$ and $p=.6$.

1) $q = 1-p = .4$ 2) $np = 150(.6) = 90$

3) $npq = 150(.6)(.4) = 36$ 4) $\sqrt{npq} = \sqrt{36} = 6$

5) $P(x=100)$ = binompdf(150, .6, 100) = .017

6) $P(x \le 100)$ = binomcdf(150, .6, 100) = .961

7) $P(x \ge 100) = 1 - P(x \le 99)$

we don't want | 99 | 100 we want — Total Prob. = .056

$= 1 - \text{binomcdf}(150, .6, 99)$

Oct 27-11:37 AM

400 voters are randomly selected.
Prob. of voting yes on Prop. 50 is .8.

1) $n = 400$ 2) $p = .8$ 3) $q = .2$

4) $np = 320$ 5) $npq = 64$

6) $\sqrt{npq} = \sqrt{64} = 8$

P(exactly 300 voters vote yes)

$P(x = 300)$ = binompdf(400, .8, 300)

$= .002$

P(at most 325 voters vote yes)

$P(x \le 325)$ = binomcdf(400, .8, 325)

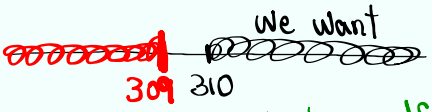
$= .752$

Oct 27-11:47 AM

P(at least 310 voters vote Yes)

Total Prob.

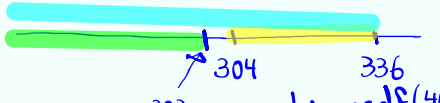
$P(x \geq 310) = 1 - P(x \leq 309)$



$= 1 - \text{binomcdf}(400, .8, 309) = .904$

P(# of Yes voters is between 304 and 336, inclusive)

$P(304 \leq x \leq 336) = P(x \leq 336) - P(x \leq 303)$



$= \text{binomcdf}(400, .8, 336) -$

$\text{binomcdf}(400, .8, 303)$

$= .961$

Oct 27-11:56 AM

Working with binomial Prob. dist:

Mean	$\mu = np$
Variance	$\sigma^2 = npq$
Standard Deviation	$\sigma = \sqrt{\sigma^2}$

Consider tossing a fair coin 100 times and Success is to land tails.

1) $n = 100$ 2) $p = .5$ 3) $q = .5$

4) $\mu = np = 50$ 5) $\sigma^2 = npq = 25$ 6) $\sigma = \sqrt{\sigma^2} = 5$

7) Usual Range (95% Range) $= \mu \pm 2\sigma = 50 \pm 2(5)$

$\Rightarrow$ 40 to 60

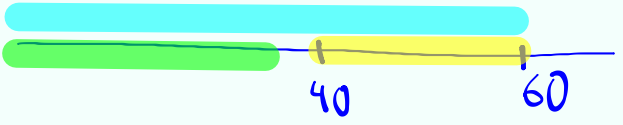
Oct 27-12:05 PM

8) P(# of tails is between 40 & 60, inclusive)

$P(40 \le x \le 60)$

Reduce by 1

= binomcdf(100, .5, 60) − binomcdf(100, .5, 39)



= .965

Oct 27-12:11 PM

Consider binomial Prob. dist. with $n=150$ and $P=.6$.

1) $q = 1-P = .4$ 2) $\mu = np = 90$

3) $\sigma^2 = npq = 36$ 4) $\sigma = \sqrt{\sigma^2} = 6$

5) 68% Range $= \mu \pm \sigma = 90 \pm 6 \Rightarrow$ 84 to 96

keep

6) $P(84 \le x \le 96)$ = binomcdf(150, .6, 96) −

binomcdf(150, .6, 83)

Reduce by 1

= .721

Oct 27-12:15 PM

You are taking a multiple-choice exam with 80 questions.

Each question has 5 choices with only one correct choice.

You are making random guesses.

1) $n = 80$ 2) $p = \frac{1}{5} = .2$ 3) $q = \frac{4}{5} = .8$

4) $\mu = np = \boxed{16}$ 5) $\sigma^2 = npq = \boxed{12.8}$

6) $\sigma = \sqrt{\sigma^2} = \sqrt{12.8} = 3.578 \approx \boxed{4}$

Usual Range $\mu \pm 2\sigma = 16 \pm 2(4) \Rightarrow \boxed{8 \text{ to } 24}$

P(# of correct guesses falls between 8 & 24, inclusive)

$P(8 \le x \le 24) =$ (Reduce by 1)

binomcdf(80, .2, 24) - binomcdf(80, .2, 7)

$= \boxed{.983}$

SG 16

Oct 27-12:22 PM